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Analysis of Asymmetric Coupled Striplines

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***Abstract*—A unified method for the quasi-static and the hybrid-mode formulation of asymmetric coupled striplines is presented. Variational expressions are derived for the matrix elements, which describe the quasi-static characteristics, for the first time. A very accurate numerical method is shown and some numerical examples are presented for the different types of asymmetric coupled striplines with anisotropic substrates.**

I. Introduction

Asymmetric coupled striplines have received considerable attention in recent years [1]–[7]. When these lines are used as building blocks for filters and directional couplers, they can provide additional advantages compared with the symmetric cases because of their impedance transform nature and flexibility.

Although accurate analytical methods of symmetrical cases are available for different types of coupled striplines [8], [9] based on the quasi-static and hybrid-mode formulations, fewer techniques have been reported for only coupled microstrips on isotropic substrates [1], [3]. Most of them are the quasi-static case; however, the variational expression for the quasi-static parameters have not been derived. In this paper, a unified analytical method is presented for the asymmetric coupled striplines. It gives the variational expressions for the matrix elements which describe the quasi-static characteristics and also it gives the hybrid-mode analysis for the frequency-dependent solutions. The formulation is general enough to treat coupled microstrips, strips with overlay, and suspended strips on anisotropic substrates. Some numerical examples are presented for the quasi-static and frequency-dependent cases.

II. Analytical Method

Fig. 1 shows the general structure of asymmetric coupled striplines with multilayered uniaxially anisotropic media, whose permittivity dyadics are

$$\bar{\bar{\epsilon}}_i = \epsilon_0(\epsilon_{i\perp}\hat{x}_0\hat{x}_0 + \epsilon_{i\perp}\hat{y}_0\hat{y}_0 + \epsilon_{i\parallel}\hat{z}_0\hat{z}_0). \qquad (1)$$

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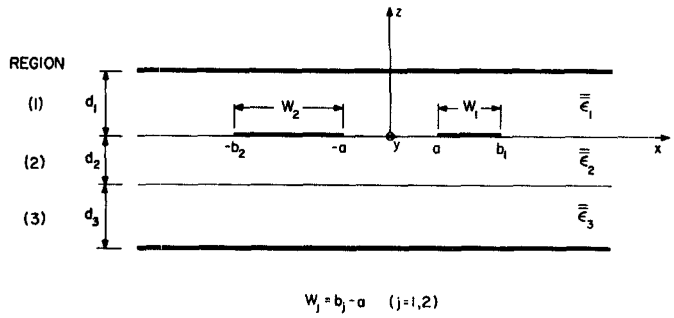

Fig. 1. General structure of the asymmetric coupled striplines.

The electric fields in each region can be expressed in terms of current density on the strips $\hat{i}(x')$ by using the extended version of the method in [9]

$$\hat{E}^{(i)}(x,y,z) = -\int_{x'}\int_{z'} \bar{\bar{Z}}^{(i)}(x,z|x',z')\cdot\hat{i}(x')\delta(z')\,dz'\,dx' \cdot e^{-j\beta_0 y}, \quad (i=1,2,3) \qquad (2)$$

where β_0 is the propagation constant in the y-direction and the dyadic Green's function $\bar{\bar{Z}}^{(i)}$ is given by

$$\bar{\bar{Z}}^{(i)}(x,z|x',z') = \frac{1}{2\pi}\sum_{l=1}^{2}\int_{-\infty}^{\infty}\left[\frac{\hat{K}_l}{K^2}Z_l^{(i)}(z|z') - \frac{\hat{z}_0}{\omega\epsilon_0\epsilon_{i\parallel}}\delta_{l1}T_l^{(i)}(z|z')\right]\cdot\hat{K}_l e^{j\alpha(x'-x)}\,d\alpha$$

$$\hat{K}_1 = \hat{x}_0\alpha + \hat{y}_0\beta_0$$

$$\hat{K}_2 = \hat{K}_1 \times \hat{z}_0$$

$$K = \sqrt{\alpha^2+\beta_0^2}. \qquad (3)$$

The scalar Green's function $Z_l^{(i)}, T_l^{(i)}$ in (3) can be derived by applying the conventional circuit theory to the equivalent circuits shown in Fig. 2 [9]. The field representation (2) is exact and it gives the basis for both the quasi-static and the hybrid-mode formulations.

A. Variational Expressions for the Quasi-Static Parameters

The capacitance matrix has been used to describe the quasi-static characteristics of asymmetric coupled lines [1], [2] and it is defined as

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} C_1 & -C_m \\ -C_m & C_2 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad (4)$$

where V_1 and Q_1 are the potential and the total charges on the right strip ($a < x < b_1$), and V_2 and Q_2 are those on the left strip ($-b_2 < x < -a$), and C_1, C_2, and C_m are the self and mutual capacitances. However, to the authors' knowledge, the variational expressions for these matrix elements have not been derived even for the coupled microstrips with the isotropic substrate. In what follows, the compliance matrix, the inverse matrix of the capacitance matrix, is introduced to describe the quasi-static characteristics of the asymmetric coupled striplines, and the variational expressions for the matrix elements are derived for the general structure shown in Fig. 1.

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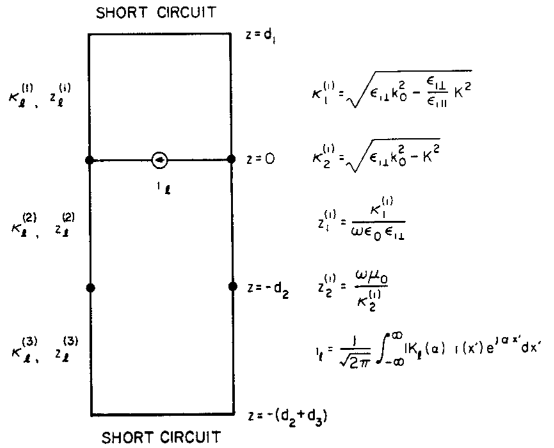

Fig. 2. Equivalent transmission-line circuits for general structure of Fig. 1.

We define the compliance matrix as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} D_1 & D_m \\ D_m & D_2 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}. \tag{5}$$

The variational expression for the self and mutual compliances D_1, D_2, and D_m are derived in the following.

In the quasi-static approximation ($\beta_0 \to 0$, $\omega \to 0$), the longitudinal electric field (2) becomes

$$E_z^{(1)}(x,z) = \frac{1}{\pi\epsilon_0}\int_{-\infty}^{\infty}\int_0^{\infty} F(\alpha)\, p_1 \frac{\cosh\{p_1(z-d_1)\alpha\}}{\sinh(p_1 d_1 \alpha)}\sigma(x') \cdot \cos\alpha(x'-x)\, d\alpha\, dx' \tag{6}$$

in region (1), where

$$F(\alpha) = \frac{1}{\epsilon_{1e}\coth(p_1 d_1 \alpha) + \epsilon_{2e} L} \tag{7}$$

$$L = \frac{1 + \frac{\epsilon_{2e}}{\epsilon_{3e}}\tanh(p_2 d_2 \alpha)\tanh(p_3 d_3 \alpha)}{\tanh(p_2 d_2 \alpha) + \frac{\epsilon_{2e}}{\epsilon_{3e}}\tanh(p_3 d_3 \alpha)} \tag{8}$$

$$p_i = \sqrt{\frac{\epsilon_{i\perp}}{\epsilon_{i\parallel}}} \qquad \epsilon_{ie} = \sqrt{\epsilon_{i\perp}\epsilon_{i\parallel}} \tag{9}$$

and $\sigma(x)$ is the charge distribution on the strips. The potential distributions at the $z = 0$ plane are given by

$$\phi(x) = \int_0^{d_1} E_z^{(1)}(x,z)\, dz = \int_{-\infty}^{\infty}\int_0^{\infty} G(\alpha; x|x')\sigma(x')\, d\alpha\, dx' \tag{10}$$

where

$$G(\alpha; x|x') = \frac{1}{\pi\epsilon_0}\cdot\frac{F(\alpha)}{\alpha}\cos\alpha(x'-x). \tag{11}$$

$\phi(x)$ should be constant over the strip conductors

$$\phi(x) = V_1 \qquad (a < x < b_1)$$
$$= V_2 \qquad (-b_2 < x < -a). \tag{12}$$

We consider the following sets of excitation:

i) $Q_1 = Q_1 \quad Q_2 = 0$ (13)

ii) $Q_1 = 0 \quad Q_2 = Q_2$ (14)

iii) $Q_1 = Q_2$. (15)

From (10), (12), and (13), we obtain

$$V_1 Q_1 = V_1 \int_a^{b_1} \sigma(x)\, dx = \int_a^{b_1}\int_{-\infty}^{\infty}\int_0^{\infty}\sigma(x) G(\alpha; x|x')\sigma(x')\, d\alpha\, dx'\, dx \tag{16a}$$

and

$$0 = V_2 \int_{-b_2}^{-a}\sigma(x)\, dx = \int_{-b_2}^{-a}\int_{-\infty}^{\infty}\int_0^{\infty}\sigma(x) G(\alpha; x|x')\sigma(x')\, d\alpha\, dx'\, dx \tag{16b}$$

by utilizing

$$\int_{-b_2}^{-a}\sigma(x)\, dx = Q_2 = 0. \tag{17}$$

Therefore, we get

$$D_1 = \left.\frac{V_1}{Q_1}\right|_{Q_2=0} = \frac{\int\int_{-\infty}^{\infty}\int_0^{\infty}\sigma(x) G(\alpha; x|x')\sigma(x')\, d\alpha\, dx'\, dx}{\left\{\int_a^{b_1}\sigma(x)\, dx\right\}^2}. \tag{18}$$

Similarly, from (10), (12), and (14), we get

$$D_2 = \left.\frac{V_2}{Q_2}\right|_{Q_1=0} = \frac{\int\int_{-\infty}^{\infty}\int_0^{\infty}\sigma(x) G(\alpha; x|x')\sigma(x)\, d\alpha\, dx'\, dx}{\left\{\int_{-b_2}^{-a}\sigma(x)\, dx\right\}^2} \tag{19}$$

$$Q_1 = \int_a^{b_1}\sigma(x)\, dx = 0 \tag{20}$$

and from (10), (12), and (15) we get

$$D_1 + D_2 + 2D_m = \frac{\int\int_{-\infty}^{\infty}\int_0^{\infty}\sigma(x) G(\alpha; x|x')\sigma(x')\, d\alpha\, dx'\, dx}{\left\{\int_a^{b_1}\sigma(x)\, dx\right\}^2} \tag{21}$$

$$\int_a^{b_1}\sigma(x)\, dx = \int_{-b_2}^{-a}\sigma(x)\, dx. \tag{22}$$

It is verified that the expressions for the compliance matrix elements (18), (19), and (21) have stationary properties, they give the upper bounds to the exact values, and that they suggest the transformation from the anisotropic problem ($\epsilon_{i\perp}, \epsilon_{i\parallel}, d_i$) to the isotropic problem ($\epsilon_{ie}, p_i d_i$), which has been obtained in other structures [8]–[10].

The Ritz procedure is applied to the variational expressions to obtain the numerical results. The effective dielectric constants for two fundamental modes (that is, C- and Π-modes) are calculated by [4]

$$\epsilon_{\text{eff},(c,\pi)} = \Big[L_1C_1 + L_2C_2 - 2L_mC_m \pm \big((L_1C_1 - L_2C_2)^2 + 4(L_mC_1 - L_2C_m)(L_mC_2 - L_1C_m)\big)^{1/2}\Big]/2 \tag{23}$$

where L_1, L_2, and L_m are the self and mutual inductances, and they are obtained from the self and mutual compliances of the case without substrates.

B. Hybrid-Mode Analysis

The analytical method for the dispersion characteristics of the asymmetric coupled striplines is the straightforward extension of that for the symmetric case [9], and the details of the formulation procedure will not be presented here.

By applying the boundary condition at the strip conductors ($\hat{E}_t = 0$) to the electric-field representation (2), the integral equation for the unknown current density $\hat{i}(x)$ and implicitly the unknown propagation constant can be obtained. This type of integral equation can be solved by using Galerkin's procedure [9].

C. Basis Functions

The unknown quantities are expanded in terms of the appropriate basis functions $\xi_k^{(j)}(x), \eta_k^{(j)}(x)$ in the Ritz procedure for the quasi-static calculations, as well as in Galerkin's procedure for the frequency-dependent calculations

$$\sigma(x) = \sum_{k=1}^{N_1} A_k^{(1)}\xi_k^{(1)}(x) + \sum_{k=1}^{N_2} A_k^{(2)}\xi_k^{(2)}(x) \tag{24}$$

$$i_x(x) = \sum_{k=1}^{N_1} B_k^{(1)}\eta_k^{(1)}(x) + \sum_{k=1}^{N_2} B_k^{(2)}\eta_k^{(2)}(x) \tag{25}$$

$$i_y(x) = \sum_{k=1}^{N_1} C_k^{(1)}\xi_k^{(1)}(x) + \sum_{k=1}^{N_2} C_k^{(2)}\eta_k^{(2)}(x) \tag{26}$$

where $A_k^{(j)}, B_k^{(j)}, C_k^{(j)}$ are unknown constants. We adopt the following basis functions, which are similar to those used in [9] and are extended for the asymmetrical case:

$$\xi_k^{(j)}(x) = \frac{T_k\left\{\dfrac{2(x-S_j)}{W_j}\right\}}{\sqrt{1-\left\{\dfrac{2(x-S_j)}{W_j}\right\}^2}} \tag{27}$$

$$\eta_k^{(j)}(x) = U_k\left\{\frac{2(x-S_j)}{W_j}\right\} \tag{28}$$

where

$$j = 1,2 \quad S_{\substack{1\\2}} = \pm\left(a + b_{\substack{1\\2}}\right)\Big/2 \quad W_{\substack{1\\2}} = b_{\substack{1\\2}} - a.$$

III. Numerical Examples

Some preliminary computations are shown in Tables I–IV. Tables I and II show the variation of the solutions with the number of basis functions to determine N_1, N_2 required to get accurate results for the quasi-static and the hybrid-mode cases, respectively. A special case (that is, the symmetric shielded coupled striplines without substrate $W_1, = W_2$, $\epsilon_{i\perp} = \epsilon_{i\parallel} = 1$, $d_1 = d_2 + d_3$) is considered in Table III. The exact analytical values for the quasi-static parameters are available in this case [11], and the comparison with the exact value by conformal mapping is made to show the validity of the present method. Also, the values for the symmetric coupled microstrips are compared with those from [8] in Table IV. Taking these preliminary numerical results into consideration, $N_1 = N_2 = 3$ for the quasi-static and $N_1 = N_2 = 2$ for the frequency-dependent cases are used in the following calculations except for the extreme wide strip cases, where one more basis function is used to obtain accurate results.

Fig. 3 shows the variation of the quasi-static characteristics with the width ratio W_2/W_1 for the different types of coupled

TABLE I
SELF AND MUTUAL COMPLIANCES OF ASYMMETRIC COUPLED MICROSTRIPS

$\epsilon_{1\perp} = \epsilon_{1//} = 1$, $\epsilon_{2\perp} = 9.4$, $\epsilon_{2//} = 11.6$

$d_1 \to \infty$, $d_2/W_1 = 1$, $d_3 = 0$

$a/W_1 = 0.25$, $W_2/W_1 = 2$

N_1	1	2	3	4
N_2	1	2	3	4
$(D_1/\epsilon_0) \times 10^2$	4.544	4.475	4.457	4.456
$(D_2/\epsilon_0) \times 10^2$	2.959	2.936	2.922	2.922
$(D_m/\epsilon_0) \times 10^2$	0.471	0.475	0.459	0.459

TABLE II
EFFECTIVE DIELECTRIC CONSTANTS FOR THE TWO FUNDAMENTAL MODES OF ASYMMETRIC COUPLED MICROSTRIPS

$\epsilon_{1\perp} = \epsilon_{1//} = 1$, $\epsilon_{2\perp} = 9.4$, $\epsilon_{2//} = 11.6$

$d_1 \to \infty$, $d_2 = 1$[mm], $d_3 = 0$

$a = 0.25$[mm], $W_1 = 1.0$[mm], $W_2 = 2.0$[mm]

$f = 20$[GHz]

N_1	1	2	3	4
N_2	1	2	3	4
c-mode	10.401	10.552	10.475	10.474
π-mode	8.169	8.187	8.194	8.190

TABLE III
SYMMETRIC SHIELDED COUPLED STRIPLINES WITHOUT SUBSTRATE

$\epsilon_{i\perp} = \epsilon_{i//} = 1$ $(i = 1,2,3)$

$d_1/W_1 = 1$, $d_1 = d_2 + d_3$

$a/W_1 = 0.2$, $W_2 = W_1$

N_1	1	2	3	4	Conformal mapping [11]
N_2	1	2	3	4	
c_1/ϵ_0	3.881	3.955	3.960	3.960	3.960
c_m/ϵ_0	0.712	0.744	0.743	0.743	0.743

TABLE IV
LINE CAPACITANCE OF SYMMETRIC COUPLED MICROSTRIPS

$\epsilon_{1\perp} = \epsilon_{1\parallel} = 1$, $\epsilon_{2\perp} = 9.4$, $\epsilon_{2\parallel} = 11.6$, $\gamma_2 = 0$

$d_1 \to \infty$, $d_3 = 0$, $W_1/d_2 = 1$, $W_2 = W_1$, $a/W_1 = 0.2$

N_1 / N_2	1 / 1	2 / 2	3 / 3	4 / 4	Ref. [8]
c-mode (even mode)	16.112	16.347	16.375	16.376	16.37
π-mode (odd mode)	25.295	26.005	26.032	26.034	26.04

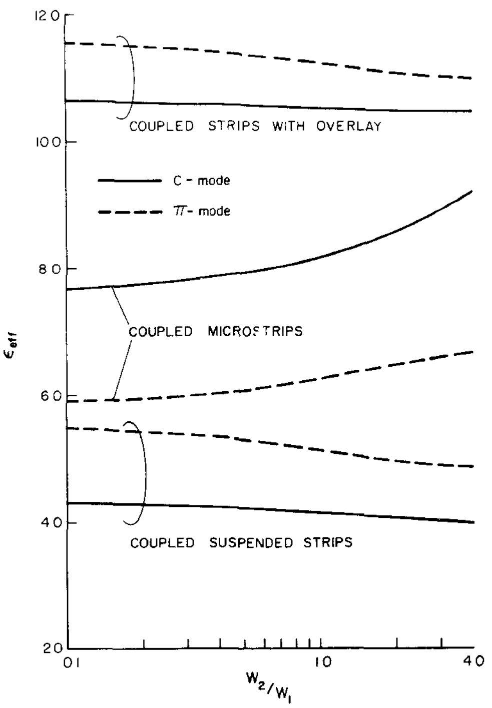

Fig. 3. Quasi-static characteristics versus width ratio W_2/W. $a/W_1 = 0.25$. Coupled microstrips:

$$\epsilon_{1\perp} = \epsilon_{1\parallel} = 1 \quad \epsilon_{2\perp} = 9.4 \quad \epsilon_{2\parallel} = 11.6$$

$$d_1 \to \infty \quad d_2/W_1 = 1 \quad d_3 = 0.$$

Coupled strips with overlay:

$$\epsilon_{1\perp} = 9.4 \quad \epsilon_{1\parallel} = 11.6 \quad \epsilon_{2\perp} = \epsilon_{2\parallel} = 13 \quad \epsilon_{3\perp} = \epsilon_{3\parallel} = 1$$

$$d_1/W_1 = 1 \quad d_2/W_1 = 1 \quad d_3 \to \infty.$$

Coupled suspended strips:

$$\epsilon_{1\perp} = \epsilon_{1\parallel} = 1 \quad \epsilon_{2\perp} = 9.4 \quad \epsilon_{2\parallel} = 11.6 \quad \epsilon_{3\perp} = \epsilon_{3\parallel} = 1$$

$$d_1 \to \infty \quad d_2/W_1 = 1 \quad d_3/W_1 = 0.2.$$

striplines with an anisotropic sapphire substrate. The effective dielectric constants of coupled microstrips become larger as the width ratio, while those of other coupled striplines become smaller.

Fig. 4 shows the dispersion characteristics of asymmetric coupled striplines ($W_2/W_1 = 2$). The values for the symmetric case ($W_2/W_1 = 1$) are shown for comparison. Frequency-dependent hybrid-mode values converge to the quasi-static values in lower frequency ranges for all cases. The C- and Π-mode phase velocities for asymmetric coupled suspended strips become equal at some frequency in the same manner as the symmetric case, which can be utilized advantageously for the directional couplers.

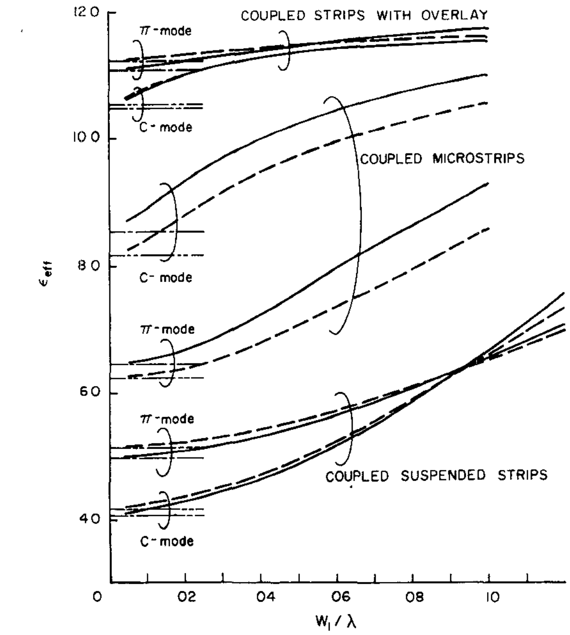

Fig. 4. Dispersion characteristics of coupled striplines.
—— Asymmetric cases (Hybrid-mode); — - — Asymmetric cases (Quasi-static): $W_2/W_1 = 2$.
- - - - Symmetric cases (Hybrid-mode); — - - — Symmetric cases (Quasi-static): $W_2/W_1 = 1$.
Dimensions are same as in Fig. 3.

IV. CONCLUSIONS

A unified analytical method is presented for the asymmetric coupled striplines, which can be applied to the quasi-static and the hybrid-mode formulations. The variational expression for the quasi-static parameters are derived for the first time. Accurate numerical results of the quasi-static and the frequency-dependent solutions are presented for the different types of coupled striplines with anisotropic dielectric substrates.

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